Number Systems

Representation of numbers of different radix.
To work Representation of numbers by using digits or other symbols (in a consistent manner).

Mainly there are four types of number systems:

(1) Decimal Number System
(2) Binary Number System
(3) Octal Number System
(4) Hexadecimal Number System.

Decimal Number System:-

(*) Contains ten unique symbols (0-9)
(*) Base (or) Radix is 10.
(*) positional Value is expressed in powers of 10
[$10, 10^2, 10^3, 10^4$ etc]

Decimal Point:-

The digits of the right of the decimal point have weights of negative powers of 10 & the digits to the left of the decimal point have weights which are positive powers of 10.

Binary Number Systems:-

(*) It has two Independent symbols (0 & 1)
(*) Base (or) Radix is 2.

A binary digit is called bit.

Binary Point:-

Separates the integer and fraction parts.

(*) The leftmost digit in any Number representation is called MSD.
(*) The rightmost digit in any Number representation is called LSD.

Octal Number System:-

- (*) 8 Independent symbols (0-7)
- (*) Base (or) Radix is 8. [(2^3)]
- (*) 3-bit group of binary can be represented by an Octal digit.

Hexadecimal Number System:- (or Hex)

- (*) 16 Independent symbols (0-9 & A-F)
- (*) Base (or) Radix is 16. [(2^4)]
- (*) 4-bit group of binary one hexadecimal digit
- (*) A 4-bit group is called <u>nibble</u>

Decimal	Binary	Octal	Hexa	Decimal	Binary	Octal	Hexa
0	00	0	0	13	1101	15	D
1	01	1	1	14	1110	16	E
2	10	2	2	15	1111	17	F
3	11	3	3	16	10000	20	10
4	100	4	4	17	10001	21	11
5	101	5	5	18	10010	22	12
6	110	6	6	19	10011	23	13
7	111	7	7	20	10100	24	14
8	1000	10	8	21	10101	25	15
9	1001	11	9	22	10110	26	16
10	1010	12	A	23	10111	27	17
11	1011	13	B	24	11000	30	18
12	1100	14	C	25	11001	31	19

Conversion of Numbers from One radix to another radix:-

① Decimal - Binary Conversion:-

We divide the given decimal number successively by 2 and read the remainders upwards to get the equivalent binary number.

Convert 52_{10} to binary (Integer part)

$$
\begin{array}{r|l}
2 & 52 \\
2 & 26 - 0 \\
2 & 13 - 0 \\
2 & 6 - 1 \\
2 & 3 - 0 \\
 & 1 - 1
\end{array}
$$

$$52_{10} = (110100)_2$$

Multiply the given fraction by 2. Keep the integer in the product as it is and multiply the new fraction in the product by 2. Continue this process and read the integers in the products from top to bottom.

Convert 0.75_{10} to binary

$0.75 \times 2 = 1.50 \rightarrow 1$

$0.50 \times 2 = 1.00 \rightarrow 1$

$$0.75_{10} = (0.11)_2$$

Convert 105.15_{10} to binary

```
2 | 105
2 | 52 - 1
2 | 26 - 0
2 | 13 - 0
2 | 6 - 1
2 | 3 - 0
    1 - 1
```

1101001

$0.15 \times 2 = 0.30 \rightarrow 0$

$0.30 \times 2 = 0.60 \rightarrow 0$

$0.60 \times 2 = 1.20 \rightarrow 1$

$0.2 \times 2 = 0.4 \rightarrow 0$

$0.4 \times 2 = 0.8 \rightarrow 0$

$0.8 \times 2 = 1.6 \rightarrow 1$

0.60×2

001001

$$105.15_{10} = 1101001.001001_2$$

Convert 6782.375_{10} to $(\ \ )_2$?

```
2 | 6782
2 | 3391 - 0
2 | 1695 - 1
2 | 847 - 1
2 | 423 - 1
2 | 211 - 1
2 | 105 - 1
2 | 52 - 1
2 | 26 - 0
2 | 13 - 0
2 | 6 - 1
2 | 3 - 0
    1 - 1
```

$0.375 \times 2 = 0.750 \rightarrow 0$

$0.75 \times 2 = 1.50 \rightarrow 1$

$0.5 \times 2 = 1.00 \rightarrow 1$

$$6782.375_{10} = 1101001111110.011_2$$

② Decimal to Octal Conversion :-

Convert 378.93_{10} to Octal

$$\begin{array}{l|l} 8 & 378 \\ \hline 8 & 47 - 2 \\ \hline 8 & 5 - 7 \end{array}$$

572

$0.93 \times 8 = 7.44 \rightarrow 7$

$0.44 \times 8 = 3.52 \rightarrow 3$

$0.52 \times 8 = 4.16 \rightarrow 4$

$0.16 \times 8 = 1.28 \rightarrow 1$

7341

$378.93_{10} = 572.7341_8$

Convert 5497_{10} to $(\quad)_8$?

$$\begin{array}{l|l} 8 & 5497 \\ \hline 8 & 687 - 1 \\ \hline 8 & 85 - 7 \\ \hline 8 & 10 - 5 \\ \hline & 1 - 2 \end{array}$$

12571

$5497_{10} = 12571_8$

Convert 6782.375_{10} to Octal

$$\begin{array}{l|l} 8 & 6782 \\ \hline 8 & 847 - 6 \\ \hline 8 & 105 - 7 \\ \hline 8 & 13 - 1 \\ \hline & 1 - 5 \end{array}$$

$0.375 \times 8 \Rightarrow 3.000 \rightarrow 3$

$6782.375_{10} = 15176.3_8$

Convert 11388.69_{10} to Octal

$$\begin{array}{l|l} 8 & 11388 \\ \hline 8 & 1423 - 4 \\ \hline 8 & 177 - 7 \\ \hline 8 & 22 - 1 \\ \hline & 2 - 6 \end{array}$$

$0.69 \times 8 = 5.52 \rightarrow 5$

$0.52 \times 8 = 4.16 \rightarrow 4$

$0.16 \times 8 = 1.28 \rightarrow 1$

$0.28 \times 8 = 2.24 \rightarrow 2$

$11388.69_{10} = 26174.5412_8$

③ Decimal to Hexadecimal :-

Convert 2598.675_{10} to hex

$$\begin{array}{l|l} 16 & 2598 \\ \hline 16 & 162 - 6 \\ \hline & 10 - 2 \end{array}$$

10 = A

A26

$0.675 \times 16 = 10.8 \rightarrow 10\,(A)$

$0.800 \times 16 = 12.8 \rightarrow 12\,(C)$

$0.8 \times 16 = 12.8 \rightarrow 12\,(C)$

$0.8 \times 16 = 12.8 \rightarrow 12\,(C)$

Accc.

$2598.675_{10} = A26.Accc_{16}$

Convert 49056_{10} to $(\quad)_{16}$?

$$\begin{array}{l|lll} 16 & 49056 & & \\ \hline 16 & 3066 & - 0 & 0 \\ \hline 16 & 191 & - 10 & A \\ \hline & 11 & - 15 & F \end{array}$$

B

$49056_{10} = BFA0_{16}$

Convert 11388.69_{10} to $(\ )_{16}$?

$$\begin{array}{l|l} 16 & 11388 \\ \hline 16 & 711 - C^{(12)} \\ \hline 16 & 44 - 7 \\ \hline & 2 - C^{(12)} \end{array}$$

$0.69 \times 16 = 11.04 \rightarrow 11 \quad B$

$0.04 \times 16 = 0.64 \rightarrow 0 \quad 0$

$0.64 \times 16 = 10.24 \rightarrow 10 \quad A$

$11388.69_{10} = 2C7C.B0A_{16}$

Convert 6782.375_{10} to $(\ )_{16}$?

$$\begin{array}{l|l} 16 & 6782 \\ \hline 16 & 423 - E^{14} \\ \hline 16 & 26 - 7 \\ \hline & 1 - 10_{A} \end{array}$$

$0.375 \times 16 = 6.0$

$6782.375_{10} = 1A7E.6_{16}$

④ <u>Binary to Decimal</u> :-

Convert 10101_2 to decimal

10101

$(1\times2^4) + (0\times2^3) + (1\times2^2) + (0\times2^1) + (1\times2^0)$

$= 2^4 + 2^2 + 2^0$

$= 16 + 4 + 1$

$= 21$

$10101_2 = 21_{10}$

Convert 11011.101_2 to decimal

11011.101

$(1\times2^4) + (1\times2^3) + (0\times2^2) + (1\times2^1) + (1\times2^0) + (1\times2^{-1}) + (0\times2^{-2}) + (1\times2^{-3})$

$= 2^4 + 2^3 + 2^1 + 2^0 + 2^{-1} + 2^{-3}$

$= 16 + 8 + 2 + 1 + 0.5 + 0.125$

$= 27.625$

$11011.101_2 = 27.625_{10}$

⑤ Octal to Decimal :-

Convert 4057.06_8 to $(\quad)_{10}$?

4057.06_8

$4\times8^3+0\times8^2+5\times8^1+7\times8^0+0\times8^{-1}+6\times8^{-2}$

$= 4\times512+40+7+0.0937$

$= 2048+40+7+0.0937$

$= 2095.0937$

$(4057.06)_8 = (2095.0937)_{10}$

Convert 7065.32_8 to decimal

$7065.32 \Rightarrow 7\times8^3+0\times8^2+6\times8^1+5\times8^0+3\times8^{-1}+2\times8^{-2}$

$= 7\times512+48+5+0.375+0.031$

$= 3584+53+0.406 \quad = 3637.406$

$(7065.32)_8 = (3637.406)_{10}$

⑥ Hexa to Decimal :-

Convert $5C7_{16}$ to decimal

$5C7_{16} = 5\times16^2+C\times16^1+7\times16^0$

$= 5\times256+12\times16+7\times1$

$= 1280+192+7 \quad = 1479$

$5C7_{16} = 1479_{10}$

Convert $A0F9.0EB_{16}$ to decimal.

$A0F9.0EB_{16} = A\times16^3+0\times16^2+F\times16^1+9\times16^0+0\times16^{-1}+E\times16^{-2}+B\times16^{-3}$

$= 10\times4096+15\times16+9\times1+14\times16^{-2}+11\times16^{-3}$

$= 40960+240+9+0.0546+0.0026 = 41209.0572$

$(A0F9.0EB)_{16} = (41209.0572)_{10}$

(7) Binary to Octal :-

Convert $110101 \cdot 101010_2$ to $(\ )_8$?

110 | 101 | · | 101 | 010

6 5 · 5 2

$(110101 \cdot 101010)_2 = (65.52)_8$

Convert $10101111001 \cdot 0111_2$ to octal.

010 | 101 | 111 | 001 | · 011 | 100

2 5 7 1 · 3 4

$(10101111001 \cdot 0111)_2 = (2571 \cdot 34)_8$

(8) Octal to Binary :-

Convert $367 \cdot 52_8$ to $(\ )_2$?

3 6 7 · 5 2

011 110 111 · 101 010

$(367 \cdot 52)_8 = (011110111 \cdot 101010)_2$

Convert $6042 \cdot 36$ to binary.

6 0 4 2 · 3 6

110 000 100 010 · 011 110

$(6042 \cdot 36)_8 = (110000100010 \cdot 011110)_2$

(9) Binary to Hexadecimal :-

Convert 1011011011_2 to $(\ )_{16}$?

0010 | 1101 | 1011

2 D B

$(1011011011)_2 = (2DB)_{16}$

Convert $01011111011 \cdot 011111_2$ to hexa

0010 | 1111 | 1011 | · 0111 | 1100

2 F B · 7 C

$(01011111011 \cdot 011111)_2 = (2FB \cdot 7C)_{16}$

(10) Hexadecimal to Binary :-

Convert $4BAC_{16}$ to binary

4BAC

4 → 0100

B → 1011

A → 1010

C → 1100

$(0100101110101100)_2$

Convert $3A9E \cdot B0D_{16}$ to binary

3 A 9 E · B 0 D

0011 1010 1001 1110 · 1011 0000 1101

$(3A9E \cdot B0D)_{16} = (0011101010011110 \cdot 101100001101)_2$

(11) Octal to hexa :-

Convert 756.603_8 to $(\ )_{16}$

$(756.603)_8$

Binary 111 101 110 . 110 000 011

0001|1110|1110 . 1100|0001|1000

1 E E . C 1 8

$(756.603)_8 = (1EE.C18)_{16}$

$(1274.774)_8$

Convert to hexa

$(1274.774)_8$

0010|1011|1100 . 1111|1110|0

2 B C . F E

$(1274.774)_8 = (2BC.FE)_{16}$

(12) Hex to Octal :-

Convert $B9F.AE_{16}$ to $(\ )_8$?

Hex B9F.AE

Binary 1011 1001 1111 . 1010 1110

101|110|011|111 . 101|011|100

5 6 3 7 . 5 3 4

$(B9F.AE)_{16} = (5637.534)_8$

Convert $2BC.FE_{16}$ to Octal

Hex $2BC.FE_{16}$

Binary 0010 1011 1100 . 1111 1110

001|010|111|100 . 111|111|100

1 2 7 4 . 7 7 4

$(2BC.FE)_{16} = (1274.774)_8$

r & (r-1)'s Complements :-

The method of complements is a technique to encode a symmetric range of positive and negative number in a way that they can use same algorithm (hardware) for addition throughout the whole range.

Subtraction of any number is implemented by adding its complement.

9's & 10's complement :-

Find 9's complement of the following :-

(a) 3465

$$\begin{array}{r} 9999 \\ -\ 3465 \\ \hline 6534 \end{array}$$

(b) 782.54

$$\begin{array}{r} 999.99 \\ -\ 782.54 \\ \hline 217.45 \end{array}$$

(c) 4526.075

$$\begin{array}{r} 9999.999 \\ -\ 4526.075 \\ \hline 5473.924 \end{array}$$

Find 10's complement of the following :-

(a) 4069

$$\begin{array}{rr} & 9999 \\ & -\ 4069 \\ \hline 9\text{'s} \rightarrow & 5930 \\ & +1 \\ \hline 10\text{'s} \rightarrow & 5931 \end{array}$$

(b) 1056.074

$$\begin{array}{rr} & 9999.999 \\ & -\ 1056.074 \\ \hline 9\text{'s} \rightarrow & 8943.925 \\ & +1 \\ \hline 10\text{'s} \rightarrow & 8943.926 \end{array}$$

Find 9's & 10's complement of 012398?

$$\begin{array}{rr} & 999999 \\ & -\ 012398 \\ \hline 9\text{'s} \rightarrow & 987601 \\ & +1 \\ \hline 10\text{'s} \rightarrow & 987602 \end{array}$$

9's complement is 987601 and 10's is 987602.

9's Complement Method of Subtraction :-

(*) Obtain 9's complement of subtrahend

(*) add it to the minuend

(*) Gets Intermediate result

If carry then +ve → add carry to LSD to obtain final result.
This is called end around carry.

If no carry then -ve → Take 9's complement of intermediate result and place -ve sign in front.

ⓐ 745.81 − 436.62

minuend 745.81

Subtrahend - 436.62

9's complement of 436.62

$$\begin{array}{r} 999.99 \\ -\ 436.62 \\ \hline 563.37 \end{array}$$

Add to minuend

$$\begin{array}{r} 745.81 \\ +\ 563.37 \\ \hline (1)309.18 \end{array}$$

↳ carry

add it to LSD

$$\begin{array}{r} 309.18 \\ +1 \\ \hline 309.19 \end{array}$$

ⓑ 436.62 − 745.81

minuend 436.62

Subtrahend - 745.81

9's complement of 745.81

$$\begin{array}{r} 999.99 \\ 745.81 \\ \hline 254.18 \end{array}$$

Add to minuend

$$\begin{array}{r} 436.62 \\ +\ 254.18 \\ \hline 690.80 \end{array}$$

No carry.

9's complement of 690.80

$$\begin{array}{r} 999.99 \\ -\ 690.80 \\ \hline -309.19 \end{array}$$

(c) 215 − 155 using 9's complement

Minuend 215

Subtrahend − 155

9's complement of 155

$$\begin{array}{r} 999 \\ -155 \\ \hline 844 \end{array}$$

Add to minuend

$$\begin{array}{r} 215 \\ +844 \\ \hline 1059 \end{array}$$

1 → carry (+ve number)

Add to LSD

$$\begin{array}{r} 059 \\ +1 \\ \hline 60 \end{array}$$

(d) 841 − 983

minuend 841

Subtrahend 983

9's complement of 983

$$\begin{array}{r} 999 \\ -983 \\ \hline 16 \end{array}$$

Add to minuend

$$\begin{array}{r} 841 \\ +16 \\ \hline 857 \end{array}$$

No carry (−ve number)

9's complement of 857

$$\begin{array}{r} 999 \\ 857 \\ \hline -142 \end{array}$$

10's complement method of Subtraction :-

- Obtain 10's complement of Subtrahend.
- Add it to the minuend
- If carry, ignore carry → indicates +ve number
- The result obtained itself is final result
- If no carry → −ve number

  Do 10's complement of result and add −ve sign

(a) 2928.54 − 416.73

  2928.54
− 0416.73

10's complement of 0416.73

  9999.99
− 0416.73

  9583.26
       +1

  9583.27

Add to minuend

  2928.54
+ 9583.27

(1)2511.81

→ carry (+ve)

ignore carry

2511.81

(b) 0416.73 − 2928.54

  0416.73
− 2928.54

10's complement of 2928.54

  9999.99
− 2928.54

  7071.45
       +1

  7071.46

Add to minuend

  0416.73
+ 7071.46

  7488.19

no carry (−ve)

10's complement of 7488.19

  9999.99
− 7488.19

  2511.80
       +1

  2511.81

Binary Arithmetic :-

Addition :-

$0+0 \rightarrow 0$

$0+1 \rightarrow 1$

$1+0 \rightarrow 1$

$1+1 \rightarrow 10$ (0 with carry 1)

Add the binary numbers 1101.101 and 111.011.

$$\begin{array}{r} 1101.101 \\ +\ 0111.011 \\ \hline 10101.000 \end{array} \qquad \begin{array}{r} 13.625 \\ +\ 7.375 \\ \hline 21.000 \\ 21 \end{array}$$

Subtraction :-

$0-0 \rightarrow 0$

$1-1 \rightarrow 0$

$1-0 \rightarrow 1$

$0-1 \rightarrow 1$, with borrow 1

Subtract 111.111_2 from 1010.01_2

$$\begin{array}{r} 1010.010 \\ -\ 0111.111 \\ \hline 0010.011 \end{array} \qquad \begin{array}{r} 10.250 \\ -\ 7.875 \\ \hline 2.375 \end{array}$$

Add the binary number 11001.1 & 10011110.11

$$\begin{array}{r} 00011001.10 \\ 10011110.11 \\ \hline 10111000.01 \end{array} \qquad \begin{array}{r} 25.50 \\ +\ 158.75 \\ \hline 184.25 \end{array}$$

Subtract 11001.1 from 10011110.11

$$\begin{array}{r} 10011110.11 \\ -\ 11001.10 \\ \hline 10000101.01 \end{array} \qquad \begin{array}{r} 158.75 \\ -\ 25.50 \\ \hline 133.25 \end{array}$$

Multiplication :-

$0 \times 0 \rightarrow 0$

$0 \times 1 \rightarrow 0$

$1 \times 0 \rightarrow 0$

$1 \times 1 \rightarrow 1$

Multiply 1101_2 by 110_2

$$\begin{array}{r} 1101 \\ \times 110 \\ \hline 0000 \\ 1101 \\ 1101 \\ \hline 1001110 \\ \hline \end{array}$$

Multiply $1011 \cdot 101_2$ by $101 \cdot 01_2$

$$\begin{array}{r} 1011 \cdot 101 \\ \times 101 \cdot 01 \\ \hline 1011101 \\ 0000000 \\ 1011101 \\ 0000000 \\ 1011101 \\ \hline 111101 \cdot 00001 \\ \hline \end{array}$$

Division :-

Divide 101101_2 by 110_2

$$\begin{array}{l} 110)\,101101 \cdot 0\ (0111 \cdot 1 \\ \quad 0110 \\ \hline \quad 01010 \\ \qquad 110 \\ \hline \quad\ 01001 \\ \qquad\ 110 \\ \hline \qquad 00110 \\ \qquad\quad 110 \\ \hline \qquad\quad (0) \end{array}$$

$101101 \div 110 = \underline{111 \cdot 1}$

Divide $110101 \cdot 11_2$ by 101_2

$$\begin{array}{l} 101)\,110101 \cdot 11\ (1010 \cdot 11 \\ \quad 101 \\ \hline \quad 0110 \\ \quad\ 101 \\ \hline \quad\ 0111 \\ \qquad 101 \\ \hline \qquad 101 \\ \qquad 101 \\ \hline \qquad (0) \end{array}$$

$110101 \div 101 = \underline{1010 \cdot 11}$

Representation of Signed numbers:-

There are two ways of representing numbers-

- (*) Sign magnitude form &
- (*) Complement form < 1's complement / 2's complement

Most digital computers do subtraction by 2's complement method, but some do it by the 1's complement method.

The advantage of performing subtraction by the complement method is reduction in the hardware.

Sign-Magnitude form:-

An additional bit called the sign-bit is placed infront of the number.

Sign bit 0 = +Ve, Sign bit 1 = -Ve

Eg:-

+41

Sign bit	magnitude					
0	1	0	1	0	0	1

-41

Sign bit	magnitude					
1	1	0	1	0	0	1

Complement form:-

① If the number is +Ve, the magnitude is represented in its true binary form and a sign bit 0 is placed infront of the MSB.

② If the number is -Ve, the magnitude is represented in its 2's or 1's complement form and a sign bit 1 is placed infront of MSB.

Eg:- 51

+51 sign-magnitude form

0	1	1	0	0	1	1
S						

+51 2's comp

0	1	1	0	0	1	1
S						

+51 1's comp

0	1	1	0	0	1	1
S						

-51 sign-mag form

1	1	1	0	0	1	1
S						

-51 1's comp

1	0	0	1	1	0	0
S						

-51 2's comp

1	0	0	1	1	0	1
S						

Given Number	Signed form	1's complement	2's complement
01101	+13	+13	+13
010111	+23	+23	+23
10111	-7	-8	-9
1101010	-42	-21	-22

characteristics of the 2's complement form:-

(*) There is one unique zero. 2's complement of 0 is 0.

(*) The leftmost bit cannot be used to express a quantity. It is a sign bit. If it is 1 → -ve & 0 → +ve

(*) For n-bit word, which includes sign bit $(2^{n-1}-1)$ positive integers, (2^{n-1}) negative integers & one 0, total 2^n unique states.

Methods of Obtaining the 2's complement of a number:—

① By obtaining the 1's complement of a given number and then adding 1.

② By Subtracting the given n-bit number N from 2^n.

③ Starting at the LSB, copy down each bit upto and including the first 1 bit encountered and complementing the remaining bits.

① Express −45 in 8-bit 2's complement form?

+45 → 00101101   −45 = ?

1st Method:—

```
      00101101
      11010010
            +1
-45   11010011
```

2nd Method

```
  2^n  = 100000000
sub45  = -00101101
          011010011
           11010011
```

3rd Method

```
00101101
       ↓
11010011
```

② Express −73.75 in 12-bit 2's complement form?

+73.75 → 01001001.1100

1st Method:—

```
01001001.1100
10110110.0011
           +1
10110110.0100
```

2nd Method

```
2^n = 100000000.0000
     -01001001.1100
      010110110.0100
       10110110.0100
```

3rd Method

```
01001001.1100
          ↓↓↓
10110110.0100
```

2's Complement Arithmetic:-

① Subtract 14 from 46 using 8-bit 2's complement arithmetic

46 − 14

14 = 00001110

−14 = 11110010

46 = 00101110

\+ (−14) = +11110010

①00100000

↳ carry → ignore it

MSB is 0

So, result is +ve

⇒ +32

② Add −75 to +26 using 8-bit 2's comp. form. (26 − 75)

+75 = 01001011

−75 = 10110101

26 = 00011010

\+ (−75) = 10110101

11001111

No carry, MSB = 1

−ve number, do 2's comp

00110001

⇒ −49

③ Add −31.5 to −93.125 12-bit's. (−93.125) + (31.5)

93.125 = 01011101.0010

31.5 = 00011111.1000

−93.125 = 10100010.1110

\+ −31.5 = +11100000.1000

① 10000011.0110

↳ carry → ignore it

MSB = 1 −ve do 2's comp

01111100.1010

−124.625

④ Add 47.25 to 55.75 using 2's complement form?

47.25 = 00101111.0100

\+ 55.75 = 00110111.1100

01100111.0000

No carry MSB = 0

+ve

+103.0