Heat Transfer from extended surface

(History = Dinosaurs)

→ The Term extended surface is commonly in reference to a solid that experiences energy transfer by conduction and convection between its boundary and surroundings

→ These extendend surface are called fins

→ The Rate of HT from a surface at a temp T_s to the surrounding medium at T_∞ is Given by newton's Law of cooling as

$$\boxed{\dot{Q}_{conv} = h A_s (T_s - T_\infty)} - W$$

where
h = convective heat transfer co-efficient

A_s = surface area.

→ when T_s and T_∞ are fixed by design considerations There are only two ways to Increase the heat transfer rate

i) TO Increase the convection co-effient h

ii) TO Increase the surface Area A_s

→ Increase h may require the ~~instalation~~ instalation of pump (or) fan, replacing the existing one with Larger one, but This approach may (or) may not be practicle, then, The Alternative is to Increase the surface area by attaching to the surface extended surfaces caled fins. made of highly condective materials such as Aluminium, copper, brass Etc---.

→ Finned surfaces are used on the surface where the heat transfer co-efficient is very low.

→ Generally the fins are used on the surfaces where heat transfer coefficient is very low.

Ex:- 1) Car Radiators 2) Electrical transformers 3) Motors 4) IC Engines 5) Electronic Equipments Etc---

→ The selection of fins are made on the basis of
1) Thermal performance 2) cost 3) weight 5) Availability space 6) pressure drop Etc---

→ Finned surfaces Manufactured by Extruding, welding (or) wrapping a thin metal sheet on base area.

Note:- 1) If heat transfer co-efficient h is high, then heat transfer rate will be high and then no need of fins

2) Fins are more effective when h is less

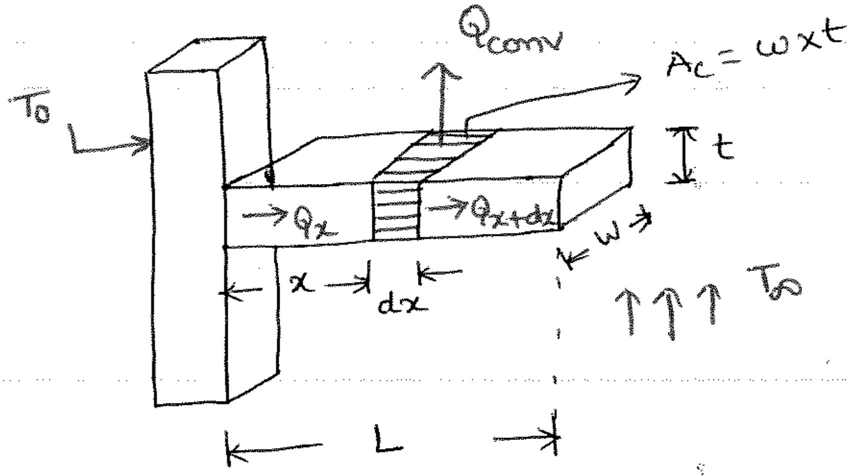
Generalized Governing equation of fin [constant Area]:-

Assumptions:-

1) Steady state Heat transfer
2) No heat generation
3) constant Thermal conductivity
4) constant heat transfer co-efficient through out
5) one Dimensional Heat conduction
6) Bonding Resistance is neglected

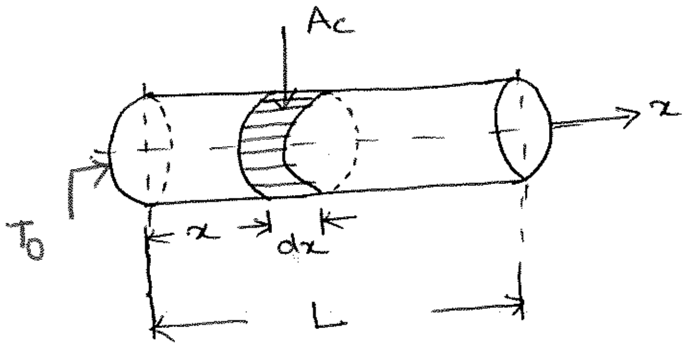
→ consider the surface of a plane wall of a temparature T_0, exposed to an ambient at T_∞

→ Let us consider a fin has constant cross section area A_c and Length L attached to the surface as shown in fig.



Temp = T(x)
perimeter (P) = 2(w+t)
cross section area $(A_c) = wt$

Temp = T(x)
perimeter $P = \pi d$
cross-sectional Area $(A_c) = \frac{\pi}{4} d^2$



→ In order to determine temparature distribution and heat flow throughout, Apply Energy balance equation

→ from Energy balance Equation

The Rate of heat conduction in to element = The Rate of Heat conduction out the element + The Rate of Heat convection from the element surface

→ The Rate of heat conduction in to Element is Q_x

→ The Rate of heat conduction out the Element is $Q_x(x+dx)$

$$\therefore Q(x+dx) = Q(x) + \frac{d}{dx}(Q(x)).dx.$$

(According to Tailor's series of approximation)

→ The Rate of Heat convection from the element surface of perimeter is Q_{conv}

$$\therefore Q_{conv} = h.dA_s[T(x) - T_\infty]$$

where dA_s = surface area of differntal Element

∴ Apply in energy balance equation

$$Q(\cancel{x}) = Q(\cancel{x}) + \frac{d}{dx}(Q(x).)dx + h\,dA_s(T(x) - T_\infty)$$

$$0 = \frac{d}{dx}\left[-KA_c\frac{dT}{dx}\right].dx + h\,dA_s(T(x) - T_\infty)$$

devide the equation with $-Kdx$

$$\frac{d}{dx}\left[A_c.\frac{dT}{dx}\right] - \frac{h\,dA_s}{K.dx}[T(x) - T_\infty] = 0$$

∵ $dA_s = p.dx$ & devided with A_c.

$$\therefore \frac{d}{dx}\left[\frac{dT(x)}{dx}\right] - \frac{h.P\cancel{dx}}{K.A_c.\cancel{dx}}[T(x) - T_\infty] = 0$$

$$\frac{d^2T}{dx^2} - \frac{hP}{KA_c}[T(x) - T_\infty] = 0 \quad \text{--- (1)}$$

where

A_c = Cross sectional area

P = perimeter

h = heat transfer co-efficient

K = Thermal conductivity of material are treated as constant (all values)

Let $\boxed{\theta(x) = T(x) - T_\infty}$, $\boxed{m^2 = \frac{hp}{KA_c}}$

Substituting these in equation (1)

$$\boxed{\frac{d^2\theta(x)}{dx^2} - m^2\theta = 0.}$$

The final sollution for above equation

$$\boxed{\theta(x) = C_1 e^{mx} + C_2 e^{-mx}}$$

(or)

$$\boxed{\theta(x) = A \cosh mx + B \sinh mx}$$

where C_1 & C_2 are evaluated by intergrating with boundary conditions specified for the fin

→ one such condition may be specified here, the Temp T_0 at the base of fin ie,

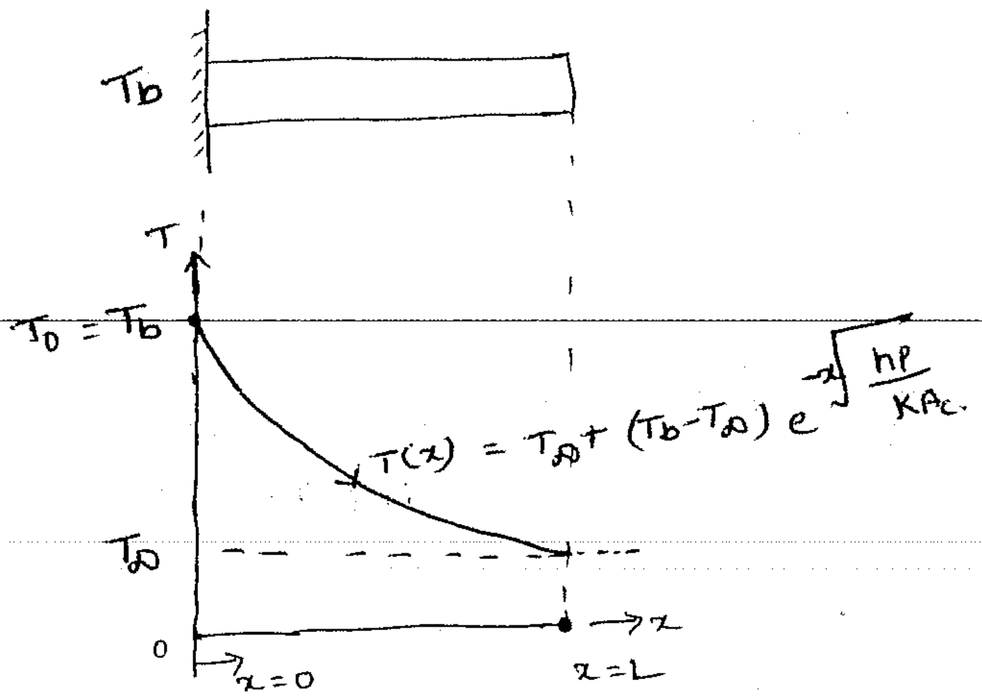
At $x = 0$, $\theta_0 = T_0 - T_\infty$

↳ This is Boundary conditions at fin Base

$$\boxed{\theta_0 = C_1 + C_2}$$

→ The secondary boundary condition specified at the fin tip, the free end of fin, that may corresponds to any four different physical situations given below.

Case 1 :- Infinite Long Fin [$T_{fin\,tip} = T_\infty$]



→ for sufficiently long fin of uniform cross section (A_c = const) the temperature of fin at the fin tip approaches the specified temperature T_∞ and thus θ approaches zero

i) Boundary condition at fin tip :-

$$\boxed{\theta(L) = T(L) - T_\infty = 0 \quad \text{as} \quad L \to \infty}$$

ii) Temperature distribution :-

$$\boxed{\frac{T(x) - T_\infty}{T_b - T_\infty} = e^{-mx} = e^{-x\sqrt{hP/KA_c}}}$$

θ/θ_b = / Temp

iii) Heat Transfer rate :-

The temparature along the fin decreases exponentially from T_b to T_∞

$$\boxed{\dot{Q}_{Long\,fin} = -KA_c \frac{dT}{dx}\Big|_{x=0} = \sqrt{hPKA_c}\,\underbrace{(T_b - T_\infty)}_{\theta_0}}$$

Case 2 :- Finite long fin and with negligible Heat loss from fin tip [Adiabatic fin tip, $\dot{Q}_{fin\,tip} = 0$]

i) Boundary condition at fin tip :-

$$\frac{d\theta}{dx}\Big|_{x=L} = 0$$

ii) Temparature distribution :-

$$\frac{T(x) - T_\infty}{T_b - T_\infty} = \frac{\cosh.m(L-x)}{\cosh.mL}$$

iii) Heat Transfer Rate :-

$$\dot{Q}_{fin,adiabatic\,tip} = -KA_c \frac{dT}{dx}\Big|_{x=0}$$

$$= -KA_c(T_b - T_\infty)$$

$$= \sqrt{hPKA_c}\,(T_b - T_\infty)\tanh.(mL).$$

Case 3 :- Specified Temperature ($T_{fin\,tip} = T_L$)

i) Boundary condition at fin tip :-

$$\theta(L) = \theta_L = T_L - T_\infty$$

ii) Temperature distribution :-

$$\frac{T(x) - T_\infty}{T_0 - T_\infty} = \frac{(T_L - T_\infty / T_0 - T_\infty) \sinh mx + \sinh m(L-x)}{\sinh mL}$$

iii) Heat transfer Rate :-

$$\dot{Q}_{fin} = -KA_c \frac{dT}{dx}\Big|_{x=0} = \sqrt{hPKA_c}\,(T_0 - T_\infty)\,\frac{\cosh mL - [T_L - T_\infty / T_0 - T_\infty]}{\sinh mL}$$

Case 4 :- Convection from Fin Tip

i) Boundary condition at fin tip :-

$$-KA_c \frac{dT}{dx}\Big|_{x=L} = hA_c[T(L) - T_\infty]$$

ii) Temperature distribution

$$\frac{T(x) - T_\infty}{T_0 - T_\infty} = \frac{\cosh m(L-x) + (h/mK)\sinh m(L-x)}{\cosh mL + (h/mK)\cdot\sinh mL}$$

iii) Convection Heat transfer Rate :-

$$\dot{Q}_{conv} = Q_{fin} = -KA_c \frac{dT}{dx}\Big|_{x=0}$$

$$= \sqrt{hPKA_c}\,(T_0 - T_\infty)\,\frac{\sinh mL + (h/mK)\cosh mL}{\cosh mL + (h/mK)\sinh mL}$$

Conventional Questions :-

i) infinite Long Fin

1) A very long fin 25mm diameter copper ($K = 380\ W/mK$) rod extends from a surface at $120^\circ C$. The temparature of surrounding air is $25^\circ C$ and the heat transfer co-efficent over the rod is $10\ W/m^2K$. calculate
   i) Heat Loss from the rod.
   ii) How long the rod should be in order to be considered infinite.

Data :-

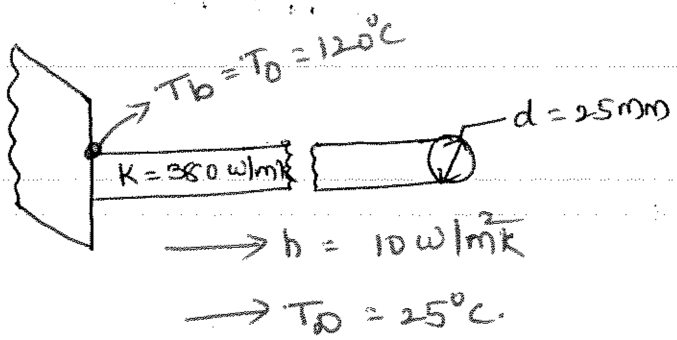
$d = 25mm = 0.025m$ $\quad K = 380\ W/mK$ $\quad T_0 = 120^\circ C$

$h = 10\ W/m^2K.$ $\quad T_\infty = 25^\circ C$

find :-

1) $Q = ?$
2) How long the rod should be inorder to be considered ($L = ?$)

Sol



1) The heat loss from the ~~fin~~ infinite long fin

$$\dot{Q}_{infinite\ fin} = \sqrt{hPKA_c}\,(T_0 - T_\infty)$$

P = perimeter $= \pi d = \pi \times 0.025$

A_c = cross sectional area $= \frac{\pi}{4} d^2$

$$\dot{Q} = \sqrt{10 \times \pi \times 0.025 \times 380 \times \frac{\pi}{4}(0.025)^2} \times (120-25)$$

$$\dot{Q} = 36.36 \text{ W}.$$

2) from finned surface of infinite long fin of boundary conditions: $T_L = T_\infty$ and no heat transfer from it's free end, because there should not be heat transfer at free end.

$$\therefore Q_{\text{infinite fin}} = Q_{\text{insulation tip fin}}$$

$$\sqrt{hP\cancel{KA_c}}\,\cancel{(T_0-T_\infty)} = \sqrt{hP\cancel{KA_c}}\,(\cancel{T_0}-T_\infty) \times \tanh \cdot mL$$

$$\tanh \cdot mL = 1$$

$$mL = 2.65$$

$$\therefore L = \frac{2.65}{m} \qquad m = \sqrt{\frac{hP}{KA_c}}$$

$$L = \frac{2.65}{2.052} \qquad = \frac{10 \times \pi \times 0.025}{380 \times \frac{\pi}{4} 0.025^2}$$

$$L = 1.289 \text{ m} \qquad m = 2.052$$

2) one end of a long rod 3cm in diameter is inserted into a furnace with the outer and projecting into the outside air. once the steady state is reached the temperature of the rod is measured at two points, 15cm apart and found to be 140°C and 100°C, when the ~~atmosphere~~ atmospheric air is at 30°C with convective coefficient of 20 W/m²·K. calculate the thermal conductivity of the rod material.

Data

$d = 3cm$ $= 0.03m$ $L = 15cm$ $= 0.15m$ $T_0 = T_b = 140°C$ $T_L = 100°C$. $T_\infty = 30°C$ $h = 20 W/m^2K$.

Find :-

1) K = ? of rod.

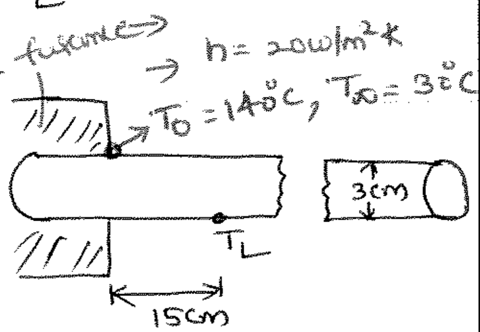
Sol for infinite long fin of temparature distribution is

$$\frac{T(x) - T_\infty}{T_0 - T_\infty} = e^{-mx}$$

at starting point $x = 0 \rightarrow T_0 = 140°C$

at 15cm ~~diston~~ apart $x = L \rightarrow T_L = 100°C$.

$$\therefore \frac{100 - 30}{140 - 30} = e^{-m \times 0.15}$$



$$m = 3.013$$

$$\because m = \sqrt{\frac{hP}{KA_c}}$$ $P = \pi d$ $A_c = \frac{\pi}{4}d^2$

$$3.013 = \sqrt{\frac{20 \times \pi \times 0.03}{K \times \frac{\pi}{4}(0.03)^2}}$$

$$K = 293.74 \ W/mK.$$

3) It is required to heat the oil to 300°C for frying purpose. A long laddle is used in frying pan. The section of the laddle is 5mm x 18mm. The surranding air is at 30°C. The Thermal conductivity of the material is 205 W/mK. If the temperature at a distance of 380 mm from the oil should not exceed 40°C. determine convective heat transfer coeffient

Data

$T_0 = 300^\circ C$, $A_c = 5 mm \times 18 mm$ $T_\infty = 30^\circ C$.

$= 90 mm^2$

$= 90 \times 10^{-6} m^2$

$K = 205 \, W/mK$, $x = 380 mm$, $T(x) = 40^\circ C = T_L$

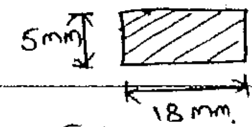
$= 0.38 m$

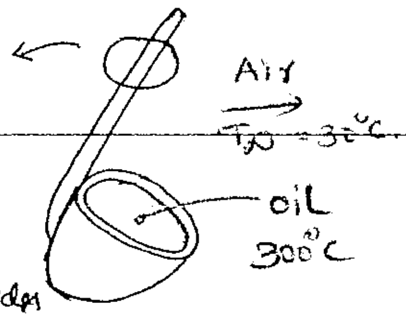
Find

1) $h = ?$

Sol The handle is treated as a very long and is infinite long fin

The temp distribution is





$$\frac{T(x) - T_\infty}{T_0 - T_\infty} = e^{-mx}$$

$A = 5 \times 18$

$P = 2(w + t)$

↳ for both sides

$P = 2(18 + 5) = 46 mm$

$= 0.046 m$

$$\frac{40 - 30}{300 - 30} = e^{-m \times 0.38}$$

$m = 8.673$

$$\because \; m = \sqrt{\frac{hP}{KA_c}}$$

$$8.673 = \sqrt{\frac{h \times 0.046}{205 \times 90 \times 10^{-6}}}$$

$h = 30.19 \, W/mK$.

ii) Adiabatic fin tip

4) An Electric motor is to be connected by horizontal steel shaft ($K = 42.56$ w/mK), 25 mm in diameter to an impeller of pump, circulating liquid metal at temperature of 540°C. If the temperature of Electric motor is Limited to a Maximum value of 52°C with the ambient air at 27°C and heat transfer coefficient of 40.7 w/m²K. what Length of shaft should be specified between the motor and pump.?

Data

$K = 42.56$ w/mK , $d = 25mm$ $= 0.025m$ $T_0 = T_b = 540°C$.

$T_L = 52°C$, $T_\infty = 27°C$, $h = 40.7$ w/m²K.

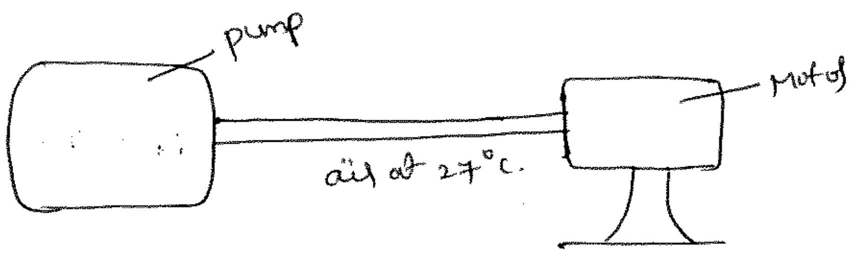
find:-

1) Length of shaft specified b/w motor and pump $(L) = ?$

sol

since one end of shaft is connected an Electric motor, thus assuming no heat Loss from the fin tip

∴ diameter of fin is very small and hence treated as fin as insulated tip (no heat) (adiabatic fin tip)



∴ from temp distribution at the fin $x = L$

$$\frac{T_L - T_\infty}{T_0 - T_\infty} = \frac{\cosh\cdot[m(L-x)]}{\cosh\cdot mL}$$

$$m = \sqrt{\frac{hP}{KA_c}} \qquad P = \pi d$$

$$A_c = \frac{\pi}{4} d^2$$

$$m = \sqrt{\frac{40.7 \times \pi \times 0.025}{42.56 \times \frac{\pi}{4}(0.025)^2}}$$

$$m = 12.37$$

$$\therefore \frac{52-27}{540-27} = \frac{\cosh\cdot[m(L-L)]}{\cosh\cdot mL}$$

$$= \frac{1}{\cosh\cdot(12.37 \times L)}$$

$$mL = 3.7139$$

$$L = \underline{\underline{0.3\,m}} = 30\,cm$$

5) Consider a stainless steel spoon ($K = 15.1\ W/mK$); partially immersed in the boiling water at 95°C in a kitchen at 25°C. The handle of the spoon has a circular cross-section 0.2cm × 1cm and it extends 18cm in the air from free surface of water. If the heat transfer coefficient on the exposed surface of spoon is $15\ W/m^2K$ calculate the temp difference across the exposed surface of the spoon handle. ~~[illegible]~~

Data

$k = 15.1\ W/mK$, $T_0 = 95^\circ C$, $T_\infty = 25^\circ C$

$A_c = 0.2\,cm \times 1\,cm$ $\quad L = 18\,cm$ $\quad h = 15\,W/m^2K$

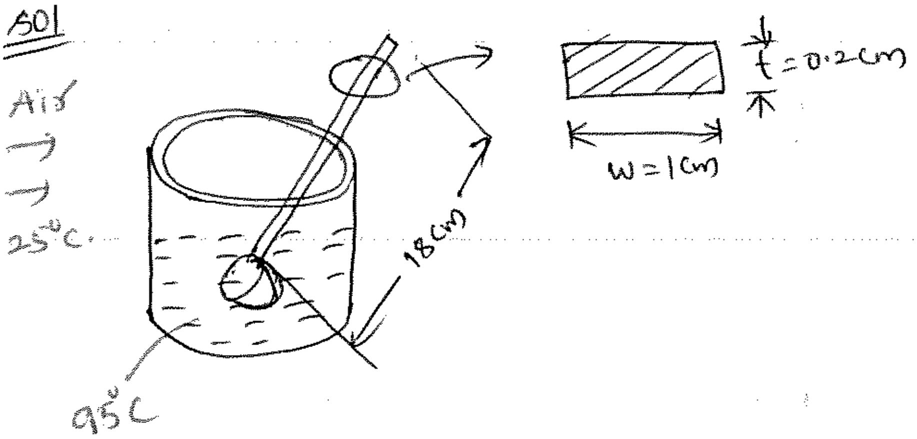
$=$ $\quad = 0.18\,m$

$=$

Find

1) Temp difference across the exposed surface of spoon handle $(T_0 - T_L) = ?$

Sol



Assume this is a short spoon with insulated tip at the end off, ∴ the temp distribution is

$$\frac{T(x) - T_\infty}{T_0 - T_\infty} = \frac{\cosh \cdot m(L-x)}{\cosh \cdot mL}$$

$$x = L$$

$$\therefore \frac{T(x) - T_\infty}{T_0 - T_\infty} = \frac{1}{\cosh \cdot mL}$$

$P = w+t \rightarrow$ single side convection

$P = 2(w+t)$ ↳ both side convection.

$$m = \sqrt{\frac{hP}{KA_c}}$$

$P = 2(w+t)$
$= 2(1+0.2)$
$= 2.4\,cm$
$= 0.024\,m$

$A_c = w \times t = 1 \times 0.2$
$= 0.2 \times 10^{-4}\,m^2$

$$\therefore m = \frac{15 \times 0.024}{15.1 \times 0.2 \times 10^{-4}}$$

$$m = 34.52$$

$$\frac{T_L - T_\infty}{T_0 - T_\infty} = \frac{1}{\cosh \cdot 34.52 \times 0.18}$$

$$\frac{T_L - 25}{95 - 25} = 3.99 \times 10^{-3}$$

$$\therefore T_L = 25.27^\circ C.$$

$\therefore$ Temp difference $T_0 - T_L = 95 - 25.27$

$$= 69.72^\circ C.$$

Corrected Fin Length :-

→ The sollution to the general fin Equation for the case of convection from fin tip is very complex to solve problems.

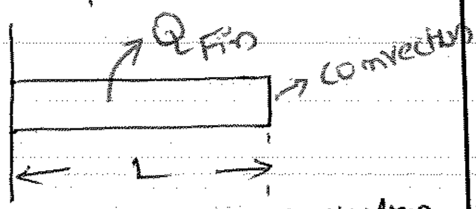
→ In order to avoid complex calculations, the heat loss from the fin tip can approximated by increasing the fin length 'δ' (Friction length) and assuming the Fin of Insulated tip

→ The fin length of convection (L) is replaced with fin with insulated fin tip and it is corrected length (Lc).

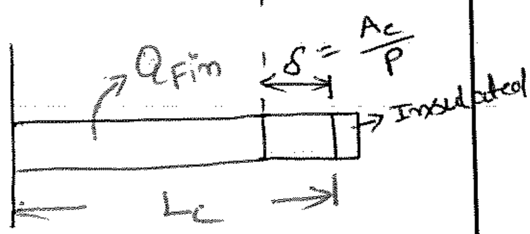
$L_c = L + \delta$

δ = Friction Length of fin

$= \frac{A_c}{P}$



Actual fin with convection at the tip



Equivalent fin with insulated tip

→ Fins subjected to ~~at their tip~~ convection at their tips can be treated as fins with insulated tip by replacing the actual fin length by the corrected length (L_c)

→ For Rectangular (or) square Fin

$$A_c = wt \quad , \quad P = \cancel{2(w+t)}\ 2w.$$

$$\therefore \delta = \frac{A_c}{P} = \frac{\cancel{w}t}{2\cancel{w}} = t/2$$

$$\therefore \delta = t/2$$

$$\therefore \boxed{L_c = L + \frac{t}{2}}$$

→ For circular Fin

$$A_c = \frac{\pi}{4} d^2 \quad , \quad P = \pi d$$

$$\therefore \delta = \frac{\frac{\pi}{4} d^2}{\pi d} \Rightarrow \delta = d/4 = r/2$$

$$\therefore \boxed{L_c = L + \frac{d}{4} = L + \frac{r}{2}}$$

→ The corrected length approximation gives very good Results when the variation of temp near the fin tip is small [near to Adiabatic].

FIN performance :-

→ The fins are used to Increase the heat transfer rate from a surface by increasing the effective surface area

→ The use of fin on a surface can not be recommended unless the increase in heat transfer, justifies the added cost and complexity associated with fins.

→ How ever, the fin itself puts a conduction resistance to heat transfer from orginal surface that [$Q\downarrow$] for this reason there is no assurance that the heat transfer rate will be increased through the use of fins

→ A parameter called as Fin effectiveness and Efficiency justifies the use of Fins, if its value is greater than unity.

→ ~~If Area increases convective~~ If fins used convective resistance decreases [$\downarrow R_{conv} = \frac{1}{hA\uparrow}$] and conductive Resistance Increases [$\uparrow R_{cond} = \frac{L\uparrow}{KA}$] so, we can not conclude the heat transfer either increases (or) decreases

→ If $L\uparrow$ $R\uparrow$ [conductive Resistance (L/KA) increased and convective Resistance decrease overally Resistance increases

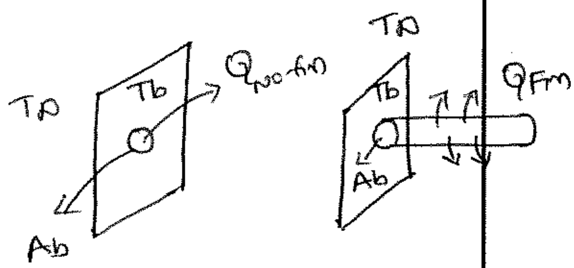
$\therefore$ ~~Q~~ $\Delta T = QR \Rightarrow Q\downarrow = \frac{\Delta T}{R\uparrow}$

1) Fin Effectiveness (ε) :-

→ To know the performance of the fin, we need to check the heat transfer with fin and heat transfer without fin.

Effectiveness

$$\boxed{\varepsilon = \frac{Q_{Fin}}{Q_{without\ fin}}}$$



→ An effectiveness of $\varepsilon_{Fin} = 1$, indicates that, the addition of fins to the surface does not effect heat transfer at all, that means no change of heat transfer even we use fin.

→ An effectiveness of $\varepsilon_{fin} < 1$, indicates that the fin actually acts as insulation, slowing down the heat transfer from the surface. This situation can occurs when fins made of low thermal conductivity.

→ An effectiveness of $\varepsilon_{fin} > 1$, indicates, that the fin enhancing heat transfer from the surface

→ However the use of fins can not be justified unless $\varepsilon_{fin} > 1$. Finned surfaces are design on the basis of Maximizing the Effectiveness ~~effectiveness~~. Efficiency for specified cost, for desired

→ For Infinite long fin of Effectiveness is

$$\varepsilon = \frac{Q_{fin}}{Q_{no\,fin}} = \frac{\sqrt{hPKA_c}\,\theta_0}{hA_c\,\theta_0} = \frac{KP}{hA_c}$$

$$\therefore \boxed{\varepsilon = \frac{KP}{hA_c}}$$

Conclusions :-

→ For better performance fin Effectiveness must be
1) Thermal conductivity of fin material must be high
2) Heat transfer co-efficient h should be very less, ie, fin are more effective in natural convection
3) $\frac{P}{A_c}$ should be very large, ie, Fins should very thin
4) Fins should be closely spaced and thin, but fins should not be too closely spaced, if they are too closely spaced the surrounding fluid may not escape easily.

2) Fin Efficiency :- [η]

→ we know that along the length of fin in the direction of heat transfer, temparature decreases and hence heat transfer also decreases as the temparature difference is less

→ The Efficiency of fin will be Maximum when the Entire fin is assume to be Maintained at constant base temparature.

→ Therefore the efficiency of fin is defined as ratio of actual heat transfer to the max heat transfer when entire fin is assume to the base temperature.

$$\boxed{\eta = \frac{Q_{fin,actual}}{Q_{max}} = \frac{Q_{fin}}{Q_{ideal}}}$$

→ If the fin efficiency is known, the heat transfer rate (Q_{fin}) throughout the fin is

$$\eta = \frac{Q_{fin}}{Q_{ideal}} \Rightarrow Q_{fin} = \eta_{fin} \times Q_{ideal}.$$

$$\therefore Q_{fin} = \eta_{fin} \times hA_{fin}(T_0 - T_\infty)$$

$$Q_{fin} = \eta_{fin} \times hPL(T_0 - T_\infty)$$

$$Q_{ideal} = hA_{fin}(T_0 - T_\infty)$$

⇓

even if ~~o~~ fin used but it was ideal

$$\boxed{Q_{nofin} = hA_{nofin}(T_0 - T_\infty)}$$

Case 1:- Efficiency of Long Fin

$$\eta = \frac{Q_{fin}}{Q_{ideal} \text{ (or) } Q_{max}} = \frac{\sqrt{hPKA_c}(T_0 - T_\infty)}{hPL(T_0 - T_\infty)} = \sqrt{\frac{KA_c}{hP}} \times \frac{1}{L}$$

→ Along ~~corr~~ cross section (A_c)

Along surface Area (PL)

$$\therefore \boxed{\eta = \frac{1}{mL}}$$

Case 2 :- Fin with Insulated Tip

$$\eta_{fin} = \frac{\sqrt{hPKA_c}\,(T_0 - T_\infty)\tanh(mL)}{hPL\,(T_0 - T_\infty)} = \frac{\tanh(mL)}{mL}$$

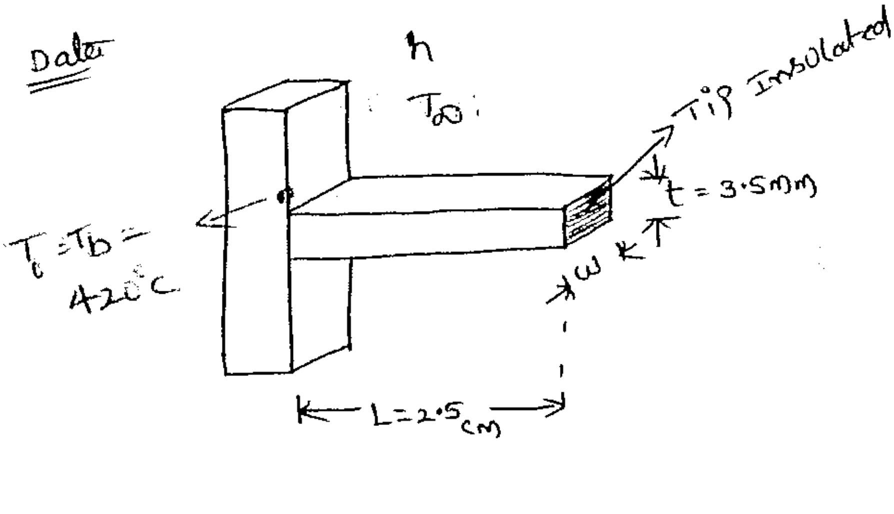
Case 3 :- $\|^{ly}$ finite long Fin with convection heat transfer at fin tip

$$\eta_{fin} = \frac{\sinh(mL) + \frac{h}{mK}(\cosh(mL))}{mL}$$

Conventional Questions :-

1) An Al alloy fin ($k = 200\ W/mK$) 3.5 mm thick and 2.5 cm long produces from a wall. The base is at 420°C and ambient air temp is 30°C. The heat transfer co-efficient may be taken as $11\ W/m^2K$. Find the heat loss and fin efficiency, if the heat loss from fin tip is negligible.

Data



$K = 200\ W/mK$

$t = 3.5mm$

$L = 2.5cm$

$T_0 = T_b = 420°C$

$T_\infty = 30°C$

$h = 11\ W/m^2K$

Find:

1) $Q_{fin} = ?$

2) $\eta_{fin} = ?$

Sol

Assume it is a short fin [Insulated fin tip, ~~bec~~ because in problem they gave heat loss from fin tip is negligible]

$$Q_{fin} = \sqrt{hPKA_c}\,(T_0 - T_\infty)\tanh . mL_c$$

Some time we take (L) (or) L_c, if we assume corrected fin length (L_c)

$$\therefore L_c = L + \frac{t}{2} \rightarrow \text{for Rectangular fin}$$

$$= 25 + \frac{3.5}{2}$$

$$L_c = 26.75\,mm = 0.02675\,mm$$

$A = w \times t$

$= 1 \times 3.5 \times 10^{-3}\ m^2$

$P = 2(w + t)$

Assume uniform width $\therefore w = 1m$

$$P = 2(1 + (3.5 \times 10^{-3}))$$

$$= 2.007\,m$$

$$m = \sqrt{\frac{hP}{KA_c}} = \sqrt{\frac{11 \times 2(1 + (3.5 \times 10^{-3}))}{200 \times (1 \times 3.5 \times 10^{-3})}}$$

$$m = 5.61$$

$$\therefore mL_c = 0.15022$$

$$Q_{fin} = \sqrt{11 \times 2.007 \times 200 \times 1 \times 3.5 \times 10^{-3}}\,(420 - 30)\tanh(0.15022) = 228.5946$$

$$\eta_{fin} = \tanh(mL_c)/mL_c = 99.25\%$$

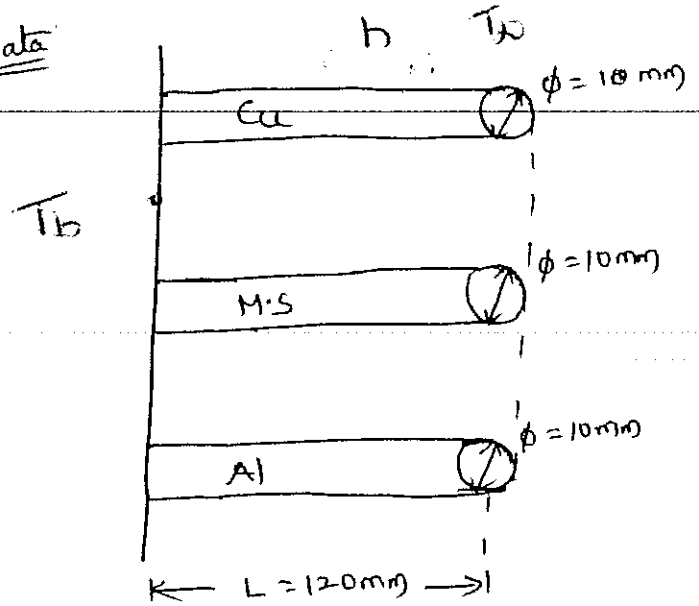
2) Three identical straight fins, 10 mm in diameter and 120 mm long are exposed to an ambient with convective heat transfer coefficient of $32\ W/m^2K$. Compare their efficiency and relative heat flow performance. The three fin materials conductivities are

$Cu = 380\ W/mK$

$Al = 210\ W/mK$

Mild steel $= 45\ W/mK$

Data



$d = 10mm$
$= 0.01m$

$L = 120mm$
$= 1.2m$

$h = 32\ W/m^2K$

$K_{cu} = 380\ W/mK$

$K_{Al} = 210\ W/mK$

$K_{ms} = 45\ W/mK$

Sol

Common configurations are

L_c, P, A_C

1) Corrected Length $(L_c) = L + \frac{d}{4}$ → for circular fins

$= 120 + \frac{\cancel{25}\ 10}{4} = 122.5$ mm
$= 1.25$m

2) perimeter $(P) = \pi d$
$= \pi \times 0.01$ m

3) Cross sectional Area $(A_C) = \frac{\pi}{4} d^2 = \frac{\pi}{4}(0.01)^2$
$= 78.53 \times 10^{-6}\ m^2$

Heat transfer

1) Copper

$$Q_{fin} = \sqrt{hPKA_c}\,(T_0 - T_\infty)$$

here $(T_0 - T_\infty)$ difference is same for all materials because base temp (T_0 (or) T_b) & ambient air temp is fixed ~~and~~, ~~transfer~~ heat transfer co-efficient h and common configuration i.e, cross sectional area A_c is also same for all materials, and hence Q_{fin} is depends ~~only~~ completely on K

$$\therefore Q_{fin} \propto K \;;\; \text{if } K\uparrow \Rightarrow Q\uparrow$$

$$\therefore \boxed{Q_{Cu\,fin} > Q_{Al\,fin} > Q_{MS\,fin}}$$

~~$Q_{Cu,fin} = hPKA_c\,Y$~~

Efficiency

$$\eta_{Cu,fin} = \frac{\tanh(mL_c)}{mL_c} \qquad m_{Cu} = \sqrt{\frac{hP}{KA_c}}$$

$$= \sqrt{\frac{32 \times 0.01\pi}{380 \times 78.53 \times 10^{-6}}}$$

$$= 13.78\,\% \qquad = 5.804$$

$$\eta_{Al,fin} = \frac{\tanh(mL_c)}{mL_c} \qquad m_{Al} = \sqrt{\frac{32 \times 0.01\pi}{210 \times 78.53 \times 10^{-6}}}$$

$$= 10.24\,\% \qquad = 7.807$$

$$\eta_{MS,fin} = \frac{\tanh(mL_c)}{mL_c} \qquad m_{MS} = \sqrt{\frac{32 \times 0.01 \times \pi}{45 \times 78.53 \times 10^{-6}}}$$

$$= 4.7\% \qquad = 16.86$$

∴ if the heat flow is more, Efficiency is also more

$$\boxed{\therefore \eta_{cu} > \eta_{Al} > \eta_{MS}}$$

∴ The Relative Heat dissipation taken with respect to copper

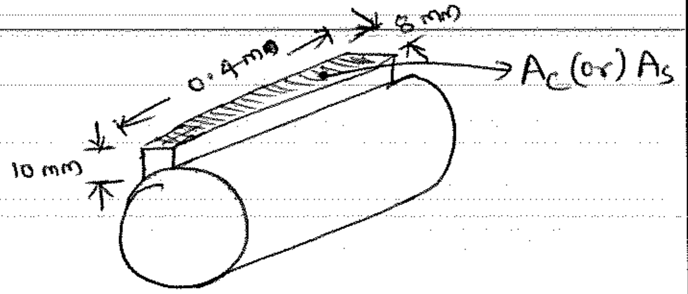
$$\text{copper} = \frac{\eta_{cu}}{\eta_{cu}} = \frac{13.78}{13.78} = 100\%$$

$$\text{Al} = \frac{\eta_{Al}}{\eta_{cu}} = \frac{10.24}{13.78} = 74.31\%$$

$$\text{MS} = \frac{\eta_{MS}}{\eta_{cu}} = \frac{4.7}{13.78} = 34.11\%$$

3) An Electric motor casing has a diameter of 0.36m and Length of 0.4m. The casing is Made of steel (K = 60 w/mK) and ~~~~ Number of fins are inserted to dissipate 400W of heat in to an ambient air, where the unit surface conductance (h) is 10 w/m²K. Each fin is to sum the entire length of ~~the~~ the motor casing. Each fin is 8mm thick and 10mm long. calculate the number of fins required to maintain the temp difference b/w casing and surrounding air of 30°C. Assume no heat Loss from the fin tip

Data



$d = 0.36m$

$L = 0.4m$

$K = 60 w/mK$

$Q_{fin} = 400 w$ (total)

$h = 10 w/m^2K$. [surface conductance, means convection, because convection takes place from surface]

$t_f = 8mm = 0.008m$

$L_f = 10mm = 0.01m$

$\Delta T = T_0 - T_\infty = 30^\circ C$.

Find:-

1) no. of fins required

Sol

$$\text{Number of fins} = \frac{Q_{total}}{Q_{single\ fin}}$$

$$Q_{single\ fin} = \sqrt{hPKA}\,(T_0 - T_\infty)\tanh(mL_c)$$

$$L_c = L + \frac{t}{2}$$

$$= 10 + \frac{8}{2} = 14mm = 0.014m$$

$$A_c = A_s = L \times t_f = 0.4 \times 0.008 = 0.0032\ m^2$$

$$P = 2(w+t) = 2(0.4 + 0.008) = 0.816m$$

$$m = \sqrt{\frac{hP}{KA}} = \sqrt{\frac{10 \times 0.816}{60 \times 0.0032}}$$

$$m = 6.519$$

$$mL_c = 6.519 \times 0.014 = 0.091$$

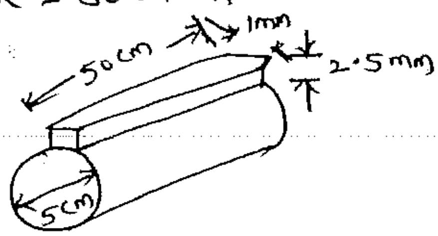
$$Q_{fin} = \sqrt{10 \times 0.816 \times 60 \times 0.032}\ (30)\ \tanh(0.91)$$

$$= 85.63\,W$$

$$\therefore \text{Number fin} = \frac{400}{85.63}$$

$$= 4.67 \cong 5 \text{ fins}$$

4) A cylinder 5cm diameter and 50cm long provided with 14 longitudinal straight fins of 1mm thick and 2.5mm height. calculate to the total heat transfer by the cylinder if the surface temperature of the cylinder is 200°C. take $h = 25\,W/m^2K$, $K = 80\,W/mK$ and ambient temp is $45^\circ C$



Data

$d = 5cm$

Length of cylinder = Length of fin = 50cm
(In this problem only) = 0.5m

Length of fin (L) = 2.5mm
(height) = 2.5×10^{-3} m

thickness of fin (t_f) = 1mm = 1×10^{-3} m

$T_0 = T_b = T_s = 200^\circ C$, $h = 25\,W/m^2K$

$K = 80\,W/mK$, $T_\infty = 45^\circ C$.

find

1) Total heat transfer rate

2) % of Increase HT Rate

3) $\eta = ?$

4) $\epsilon = ?$

Sol

If nothing is mentioned about fin, treated it as short fin of Insulated tip

$$\frac{\cancel{Q_{total}}}{\cancel{Q_{with\ fin\ area}}} \quad Q_{total} = Q_{with\ fin\ area} + Q_{with\ cylinder\ area}$$

$$Q_{with\ fins\ area} = 14 \times Q_{single\ fin}$$

$$Q_{single\ fin} = \sqrt{hPKA_c}\,(T_0 - T_\infty)\tanh(mL_c)$$

only fin data

$$L_c = L + \frac{t}{2} = 2.5 + \frac{1}{2}$$

$$= 3mm = 3\times10^{-3}m$$

$$m = \sqrt{\frac{hP}{KA_c}}, \qquad P = 2(w+t)$$

$$= 2(50+1) = 1002\,mm$$

$$= 1.002\,m$$

$$\therefore m = \sqrt{\frac{25\times1.002}{80\times0.5\times10^{-3}}} \qquad A_c = wt = 0.5\times1\times10^{-3}\,m^2$$

$$= 25.02$$

$$Q_{single\ fin} = \sqrt{25\times1.002\times80\times0.5\times10^{-3}}\times(200-45)\tanh(25.02\times3\times10^{-3})$$

$$= 11.62\ W$$

HT for 14 fin $= 14\times11.62 = 162.27\,W.$

$$Q_{with\ cylinder\ area} = Q_{convection}$$

$$= hA_s(T_0 - T_\infty)$$

$A_s = (\pi D - 14t) L_{cylinder} \times 50 \times 10^{-2}$

$Q_{cylinder\,area} = 25 \times (\pi \times 5 \times 10^{-2} - 14 \times 1 \times 10^{-3})(200 - 45)$

$= \underline{277.2 W}.$

$\therefore Q_{total} = 162.6 + 277.2 = \underline{439.9 W}$

2) % Increase in HT $= \dfrac{Q_{total\,fins} - Q_{no\,fin}}{Q_{no\,fin}}$

$Q_{no\,fin} = Q_{conv\,without\,fin}$

$= hA_s(T_0 - T_\infty)$

$= h\pi DL (T_0 - T_\infty)$

$= 25 \times \pi \times 5 \times 10^{-2} \times 50 \times 10^{-2} (200 - 45)$

$= \underline{304.34 W}$

3) $\eta_{fin} = \dfrac{Q_{fin\,actual}}{Q_{ideal}}$

$Q_{fin,actual} = 11.62 W.$

$Q_{ideal} = hA_{fin}(T_0 - T_\infty)$

$A_{fin} = PL = AC$

$= \cancel{25 \times 1.002} \times$

$Q_{ideal} = 25 \times 1.002 \times 0.5 \times 10^{-2} (200 - 45)$

$= 19.41 W$

$\therefore \eta_{fin} = \dfrac{11.62}{19.41} = 59.85\%$

4) $\varepsilon = \dfrac{Q_{fin}}{Q_{no\,fin}} = \dfrac{162.27}{304.34} = 0.533.$

$\therefore \underline{\varepsilon < 1}$